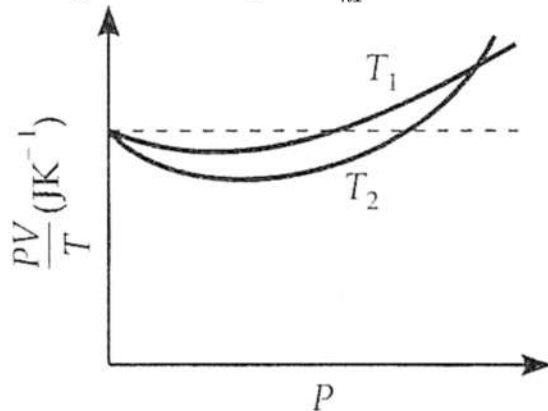


then on the surface of the earth, the acceleration due to gravity

- a) will be directed towards the centre but not the same everywhere. b) cannot be zero at any point.
 c) will have the same value everywhere but not directed towards the centre. d) will be same everywhere in magnitude directed towards the centre.

6. The figure shows the plot of $\frac{PV}{nT}$ versus P for oxygen gas at two different temperatures. [1]



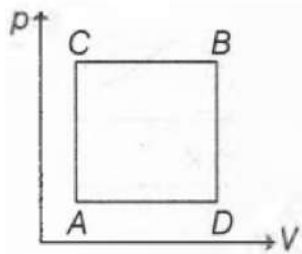
Read the following statements concerning the above curves:

- i. The dotted line corresponds to ideal gas behaviour.
 ii. $T_1 > T_2$
 iii. The value of $\frac{PV}{nT}$ at the point, where the curves meet on the Y-axis is the same for all gases.

Which of the above statements is true?

- a) none of these b) all the these
 c) (i) only d) (i) and (ii) only

7. A gas can be taken from A to B via two different processes ACB and ADB. [1]



When path ACB is used 60 J of heat flows into the system and 30 J of work is done by the system. If path ADB is used work done by the system is 10 J, the heat flow into the system in path ADB is

- a) 20 J b) 40 J
 c) 80 J d) 100 J

8. Speed of a transverse wave on a straight wire (mass 6.0 g, length 60 cm, and area of cross-section 1.0 mm^2) is 90 ms^{-1} . If Young's modulus of wire is $16 \times 10^{11} \text{ Nm}^{-2}$ the extension of wire over its natural length is: [1]

- a) 0.03 mm b) 0.01 mm
 c) 0.02 mm d) 0.04 mm

9. The pressure at a depth of h in a fluid of density ρ at a place where the acceleration due to gravity is g and the pressure at $h = 0$ is p_0 is given by: [1]

- a) $p = p_0 + \rho gh$ b) $p = p_0 + 2\rho gh$

c) Assertion is correct statement but reason is wrong statement.

d) Assertion is wrong statement but reason is correct statement.

18. **Assertion (A):** Pressure can be subtracted from pressure gradient. [1]

Reason (R): Only like quantities can be added or subtracted from each other.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

19. Time period T of a simple pendulum may depend upon its mass m , length l and acceleration due to gravity g . Find an expression for time period T by dimensional analysis method. [2]

20. A batsman hits back a ball straight in the direction of the bowler without changing its initial speed of 12 ms^{-1} . If the mass of the ball is 0.15 kg , determine the impulse imparted to the ball. (Assume linear motion of the ball) [2]

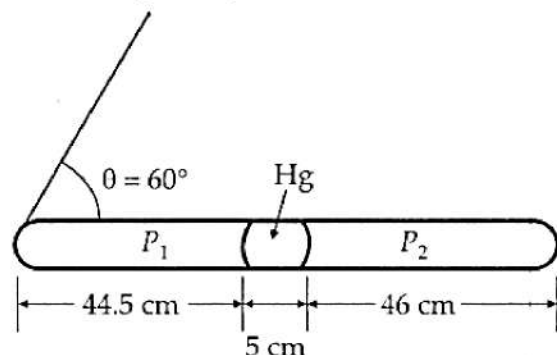
21. Out of aphelion and perihelion, where is the speed of the earth more and why? [2]

OR

Are we living at the bottom of a gravitational well? Give reason.

22. A steel wire of length 2.0 m is stretched through 2.0 mm . The cross-sectional area of the wire is 4.0 mm^2 . Calculate the elastic potential energy stored in the wire in the stretched condition. Young's modulus of steel is $2.0 \times 10^{11} \text{ Nm}^{-2}$. [2]

23. A thin tube of uniform cross-section is sealed at both ends. When it lies horizontally, the middle 5 cm length contains mercury and the two equal ends contain air at the same pressure P . When the tube is held at an angle of 60° with the vertical, then the lengths of the air columns above and below the mercury column are 46 cm and 44.5 cm respectively. [2]



Calculate the pressure P in cm of mercury. The temperature of the system is kept at 30°C .

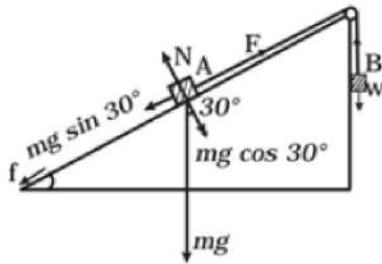
OR

What are the assumptions of kinetic theory of gas?

24. A car starting from rest accelerates at the rate f through a distance s , then continues at constant speed for sometime t and then decelerate at the rate $\frac{f}{2}$ to come to rest. If the total distance is $5s$, then prove that $s = \frac{1}{2} ft^2$. [2]

25. Block A of weight 100 N rests on a frictionless inclined plane of slope angle 30° (Fig.). A flexible cord attached to A passes over a frictionless pulley and is connected to block B of weight W . Find the weight W for which the [2]

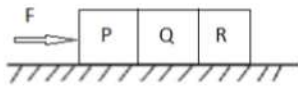
system is in equilibrium.



Section C

26. Consider a cycle tyre being filled with air by a pump. Let V be the volume of the tyre (fixed) and at each stroke of the pump $\Delta V \ll V$ of air is transferred to the tube adiabatically. What is the work done when the pressure in the tube is increased from P_1 to P_2 ? [3]

27. Three identical blocks each having a mass m , are pushed by a force F on a frictionless table as shown in figure [3]



What is the acceleration of the blocks? What is the net force on the block P? What force does P apply on Q. What force does Q apply on R?

28. A venturimeter is connected to two points in the mains where its radii are 20cm and 10cm, respectively, and the levels of water column in the tubes differ by 10cm. How much water flows through the pipe per minute? [3]

OR

Show that if n equal rain droplets falling through air with equal steady velocity of 10 cms^{-1} coalesces, the resultant drop attains a new terminal velocity of $10 n^{\frac{2}{3}} \text{ cms}^{-1}$.

29. Show that the wave obtained as a result of interference of two sinusoidal waves is also a sinusoidal wave. [3]

OR

A bat is flitting about in a cave, navigating via ultrasonic beeps. Assume that the sound emission frequency of the bat is 40 kHz. During one fast swoop directly toward a flat wall surface, the bat is moving at 0.03 times the speed of sound in air. What frequency does the bat hear reflected off the wall?

30. Two absolute scales A and B have triple points of water defined to be 200 A and 350 B. If T_A and T_B are the triple points of water on the two scales, then find out the relation between T_A and T_B (Given, triple point of water on Kelvin scale is $T_K = 273.15 \text{ K}$). [3]

Section D

31. One end of a U-tube containing mercury is connected to a suction pump and the other end to atmosphere. A small pressure difference is maintained between the two columns. Show that, when the suction pump is removed, the column of mercury in the U-tube executes simple harmonic motion. [5]

OR

Draw a graph to show the variation of P.E., K.E. and total energy of a simple harmonic oscillator with displacement.

32. State the parallelogram law of vector addition and find the magnitude and direction of the resultant of two vectors \vec{P} and \vec{Q} inclined at an angle θ with each other. What happens, when $\theta = 0^\circ$ and $\theta = 90^\circ$? [5]

OR

A vector has magnitude and direction. Does it have a location in space? Can it vary with time? Will two equal vectors a and b at different locations in space necessarily have identical physical effects? Give examples in support of your answer.

33. A solid disc and a ring, both of radius 10 cm are placed on a horizontal table simultaneously, with initial angular [5]

speed equal to $10\pi \text{ rad s}^{-1}$. Which of the two will start to roll earlier? The co-efficient of kinetic friction is $\mu_k = 0.2$.

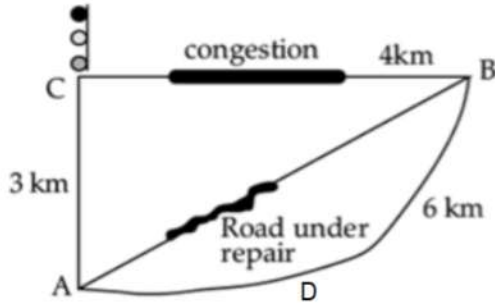
OR

Two particles each of mass m and speed v travel in opposite direction along parallel lines, separated by a distance d . Show that vector angular momentum of the two particles system is same whatever be the point about which angular momentum is taken.

Section E

34. **Read the text carefully and answer the questions:** [4]

Tabu lives at A. He was supposed to go to his uncle's house at B. A and B is connected by a straight road 5 km long. But that day the road was under repair. So, all the buses were following a diversion via C. A to B via C is 7 km. Moreover, this route is congested. There is a traffic signal at C also.



Tabu got a seat just behind the driver. He noticed that the minimum reading on the speedometer was 15 km/h. But ultimately the bus took 1 hour to reach B. He could not understand the fallacy.

- What is the distance and displacement of Tabu?
- Why the speedometer reading was minimum 15 km/h, but actual time required to cover 7 km was 1 hour?
- If the bus followed ADB path and reached B in 1 hour, find the average speed of the bus.

OR

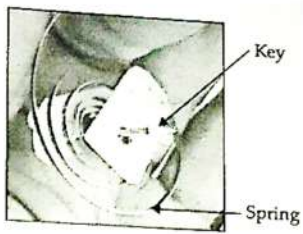
Which of the following graphs represents the motion of the bus if it covers AC distance at a speed 15 km/h and CB distance at a speed 20 km/h and total distance is covered in 1 hour including halt at traffic signal?

35. **Read the text carefully and answer the questions:** [4]

Clockwork refers to the inner workings of mechanical clock or watch (where it is known as "movement") and different types of toys which work using a series of gears driven by a spring. Clockwork device is completely mechanical and its essential parts are:

- A key (or crown) which you wind to add energy
- A spiral spring in which the energy is stored
- A set of gears through which the spring's energy is released. The gears control how quickly (or slowly) a clockwork machine can do things. Such as in mechanical clock/watch the mechanism is the set of hands that sweep around the dial to tell the time. In a clockwork car toy, the gears drive the wheels.

Winding the clockwork with the key means tightening a sturdy metal spring, called the mainspring. It is the process of storing potential energy. Clockwork springs are usually twists of thick steel, so tightening them (forcing the spring to occupy a much smaller space) is actually quite hard work. With each turn of the key, fingers do work and potential energy is stored in the spring. The amount of energy stored depends on the size and tension of the spring. Harder a spring is to turn and longer it is wound, the more energy it stores.



While the spring uncoils, the potential energy is converted into kinetic energy through gears, cams, cranks and shafts which allow wheels to move faster or slower. In an ancient clock, gears transform the speed of a rotating shaft so that it drives the second hand at one speed, the minute hand at $\frac{1}{60}$ of that speed, and the hour hand at $\frac{1}{3600}$ of that speed. Clockwork toy cars often use gears to make themselves race along at surprising speed.

- (i) What type of energy is stored in the spring while winding it?
- (ii) When the spring of a clockwork uncoils, how the potential energy of spring changes?
- (iii) Which instrument transform the speed of a rotating shaft to drive wheels slower or faster?

OR

The amount of energy stored in a spring depends on which factor?

Solution
SAMPLE PAPER - 3
Class 11 - Physics
Section A

1. (a) $[ML^2T^{-2}A^{-1}]$

Explanation: $[\phi] = [BA] = \left[\frac{F}{qv} A \right]$
 $= \left[\frac{MLT^{-2}L^2}{ATLT^{-1}} \right] = [ML^2T^{-2}A^{-1}]$

2. (d) 18.75 N m

Explanation: Given that,

$$k = 5 \times 10^3 \text{ N/m}$$

$$W = \frac{1}{2} k (x_2^2 - x_1^2)$$

$$= \frac{1}{2} \times 5 \times 10^3 [(0.1)^2 - (0.05)^2]$$

$$W = \frac{5000}{2} \times 0.15 \times 0.05$$

$$= 18.75 \text{ N-m}$$

3. (b) 1 ms^{-1}

Explanation: $K_R = \frac{1}{2} I \omega^2$, $K_T = \frac{1}{2} m v^2$

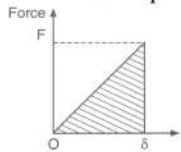
$$\therefore \frac{1}{2} m v^2 = \frac{1}{2} I \omega^2$$

$$\text{or } v = \omega \sqrt{\frac{I}{m}} = 2 \sqrt{\frac{3}{12}} = 1 \text{ ms}^{-1}$$

4. (d) work done by F is equal to $\frac{YS\delta^2}{2L}$

Explanation:

If a gradually increasing force is applied, then force-elongation curve will be a straight line as shown in figure. Work done by the force is equal to area under the curve (shown dotted).



Therefore, $W = \frac{1}{2} F \cdot \delta$

But $\delta = \frac{F}{SY} \cdot L$

or $F = \delta \frac{YS}{L}$

Hence proved. According to law of conservation of energy, strain energy stored in material of the wire is equal to work (W) done by the force (F).

5. (b) cannot be zero at any point.

Explanation: Acceleration due to gravity $g = 0$, at the centre if we assume the earth as a sphere of uniform density, then it can be treated as point mass placed at its centre. But on the surface of the earth, the acceleration due to gravity cannot be zero at any point.

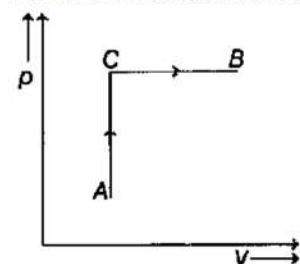
6. (b) all the these

Explanation: All three given statements are true.

7. (b) 40 J

Explanation:

For the ABC as shown in the figure below,



According to the first law of thermodynamics, heat supplied, $\Delta Q = \text{work done } (\Delta W) + \text{internal energy } (\Delta U)$

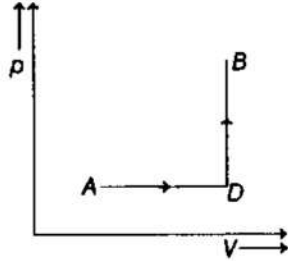
$$\Rightarrow \Delta Q_{cb} = \Delta W_{acb} + (U_B - U_A)$$

[where, $\Delta U = U_B - U_A$]

Substituting the given values,

$$U_B - U_A = 60 - 30 = 30\text{J} \dots(i)$$

Similarly for the ADB as shown in the figure below,



$$\Delta Q_{ADB} = \Delta W_{ADB} + (U_B - U_A)$$

$$\Rightarrow \Delta Q_{ADB} = 10 + 30 \text{ [using Eq. (i)]}$$

$$= 40 \text{ J}$$

8. (a) 0.03 mm

Explanation: Given, $l = 60 \text{ cm}$, $m = 6\text{g}$, $A = 1 \text{ mm}^2$, $v = 90 \text{ m/s}$ and $Y = 16 \times 10^{11} \text{ Nm}^{-2}$

$$\text{Using, } v = \sqrt{\frac{T}{m}} \times l \Rightarrow T = \frac{mv^2}{l}$$

$$\text{Again from, } Y = \frac{T}{A} \frac{\Delta L}{L_0}$$

$$\Delta L = \frac{TL}{YA} = \frac{mv^2 \times l}{l(YA)} = \frac{6 \times 10^{-3} \times 90^2}{16 \times 10^{11} \times 10^{-6}} = 3 \times 10^{-4} \text{ m} = 0.03 \text{ mm}$$

9. (a) $p = p_0 + \rho gh$

Explanation: According to Pascal's Law,

$$\text{Change in Pressure } (P - P_0) = h\rho g$$

$$\text{if } h = 0 \text{ then } P - P_0 = 0$$

$$\text{and } P = P_0$$

10. (b) $\frac{1}{R}$

Explanation: Gravitational force provides the required centripetal force.

The gravitational force provides the required centripetal force in orbit of earth.

$$\therefore \frac{GM_e M}{R^2} = \frac{mv_0^2}{R}$$

$$v_0 = \sqrt{\frac{GM_e}{R}}$$

$$\text{Kinetic energy} = \frac{1}{2}mv_0^2$$

$$\therefore KE = \frac{1}{2}m \left(\frac{GM_e}{R} \right)^{2/2}$$

$$= \frac{1}{2} \frac{mGM_e}{R}$$

$$KE \propto \frac{1}{R}$$

11. (c) 8 cm

Explanation: $I_f = I_C + mr^2$

$$mK_r^2 = mK_c^2 + mr^2$$

$$\therefore K_C = \sqrt{K_r^2 - r^2}$$

$$= \sqrt{(10)^2 - (6)^2}$$

$$= 8 \text{ cm}$$

12. (b) 900 m³

Explanation: $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

$$\frac{1 \times 500}{300} = \frac{0.5 \times V_2}{270}$$

$$V_2 = \frac{5 \times 270}{3 \times 0.5} = 900 \text{ m}^3$$

13. (b) 5

Explanation: It is given that the 3 waves arrive at a point with a successive difference of 90° .

So, the waves with amplitude 10mm and 7mm will meet out of phase and destructive super position takes place.

Hence, the amplitude of the resultant wave is $10 - 7 = 3\text{mm}$

Now, the waves of 3 mm and 4 mm are out of phase by 90°

Therefore, by using Pythagoras theorem, they will interfere with each other and new amplitude of the wave is given as,

$$A^2 = 3^2 + 4^2$$

$$A^2 = 25 \text{ or } A = 5 \text{ mm}$$

14. (d) in an isochoric process pressure remains constant

Explanation: In an isochoric process volume remains constant.

15. (a) $\frac{2Gm}{r^2} \geq c$

Explanation: The criterion for a star to be black hole is:

$$\frac{Gm}{c^2 r} \geq \frac{1}{2} \text{ or } \sqrt{\frac{2Gm}{r}} \geq c$$

16. (b) Both A and R are true but R is not the correct explanation of A.

Explanation: According to triangle law of vector addition, if three vectors \vec{A} , \vec{B} and \vec{C} are represented by three sides of a triangle taken in the same order, then their resultant is zero. The resultant of three non-coplanar vectors can never be zero.

17. (c) Assertion is correct statement but reason is wrong statement.

Explanation: Assertion is correct statement but reason is wrong statement.

18. (d) A is false but R is true.

Explanation: [Pressure gradient] = $\left[\frac{dP}{dx}\right] = \frac{[ML^{-1}T^{-2}]}{[L]} = [ML^2T^{-2}]$ which are not the dimensions of pressure, therefore pressure cannot be subtracted from a pressure gradient.

Section B

19. Time period, T of a simple pendulum depends on (i) mass, m, (ii) length, l and (iii) acceleration due to gravity, g.

We assume the relation as, $T = k m^a l^b g^c$, where k is a dimensionless constant and a, b, and c, are the exponents which are to be determined.

The dimension of time period $T = [M^0 L^0 T^1]$, mass $m = [M]$, length $l = [L]$ and acceleration due to gravity $g = [L T^{-2}]$.

Considering dimensions on both sides, we have,

$$\therefore [M^0 L^0 T^1] = [M]^a [L]^b [L T^{-2}]^c = [M^a L^{b+c} T^{-2c}]$$

On equating the dimensions on both the sides, we get,

$$a = 0 \dots(i)$$

$$b + c = 0 \dots(ii)$$

$$\text{and } -2c = 1 \dots(iii)$$

$$\Rightarrow a = 0, b = +\frac{1}{2} \text{ and } c = -\frac{1}{2}$$

Thus on substituting the values, time period is,

$$T = km^0 l^{\frac{1}{2}} g^{-\frac{1}{2}}$$

$$\text{or } T = k\sqrt{\frac{l}{g}}$$

20. Impulse = change in linear momentum

initial momentum of ball = mu

$$= 0.15 \times 12 = 1.8\text{kg.m/sec}$$

$$\text{final momentum of ball} = -mu$$
$$= -0.15 \times 12 = -1.8\text{kgm/sec}$$

now,

change in momentum of ball = final - initial

$$= -1.8 - 1.8 = -3.6\text{kgm/sec}$$

so,

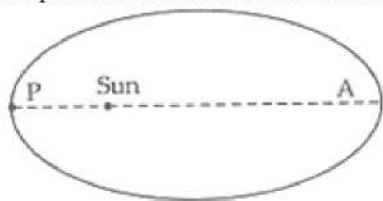
change in momentum of bat = -change in momentum of ball

$$\text{change in momentum of bat} = 3.6\text{kgm/s}$$

now,

$$\text{impulse imparted to ball} = \text{change in momentum of bat} = 3.6\text{kgm/s} = 3.6\text{Ns}$$

21. The earth revolves around the sun in an elliptical orbit or by Kepler's first law and sun remains at its one focus. The position of earth at P and A at shortest and longest distance are called perihelion and Aphelion respectively.



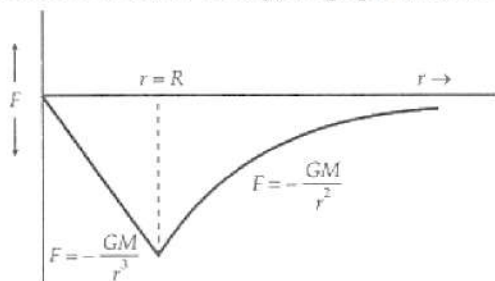
According to the second law of Kepler's the areal velocity of planet around the sun is constant.

$$\frac{dA}{dt} = \frac{L}{2m} = \frac{r \times p}{2m} = \frac{r \times mv}{2m} = \frac{1}{2} r \times v$$

Hence, if r increases at Aphelion then v decreases and vice-versa at p.

OR

Yes, we are living at the bottom of a gravitational well. Figure shows the variation of gravitational force F with distance r from the centre of the earth. Clearly, the graph has a force minimum at the surface of the earth ($r = R$).



22. The strain in the wire $\frac{\Delta l}{l} = \frac{2.0 \text{ mm}}{2.0 \text{ m}} = 10^{-3}$

The stress in the wire = $Y \times$ strain

$$= 2.0 \times 10^{11} \text{ Nm}^{-2} \times 10^{-3} = 2.0 \times 10^8 \text{ Nm}^{-2}$$

The volume of the wire = $(4 \times 10^{-6} \text{ m}^2) \times (2.0 \text{ m})$

$$= 8.0 \times 10^{-6} \text{ m}^3$$

the elastic potential energy stored

$$= \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

$$= \frac{1}{2} \times 2.0 \times 10^8 \text{ Nm}^{-2} \times 8.0 \times 10^{-6} \text{ m}^3$$

$$= 0.8 \text{ J}$$

23. Let A be the area of cross-section of the tube. When the tube is horizontal, the 5 cm column of Hg is in the middle, so length of air column on either side at pressure P

$$= \frac{46+44.5}{2} = 45.25 \text{ cm}$$

When the tube is held at 60° with the vertical, the lengths of air columns at the bottom and the top are 44.5 cm and 46 cm respectively. If P_1 and P_2 are their pressures, then

$$P_1 - P_2 = 5 \cos 60^\circ = 5 \times \frac{1}{2} = \frac{5}{2} \text{ cm of Hg}$$

Using Boyle's law for constant temperature,

$$PV = P_1V_1 = P_2V_2$$

$$P \times A \times 45.25 = P_1 \times A \times 44.5 = P_2 \times A \times 46$$

$$\therefore \frac{P \times 45.25}{44.5} = \frac{P \times 45.25}{46} = \frac{5}{2}$$

$$\text{or } P = \frac{5 \times 44.5 \times 46}{2 \times 45.25 \times 15} = 75.4 \text{ cm.}$$

OR

The assumptions of kinetic theory of gases are:-

- A gas consists of a very large number of molecules which should be elastic spheres and identical for a given gas.
 - The molecules of a gas are in a state of continuous rapid and random motion.
 - The size of gas molecules is very small as compared to the distance between them.
 - The molecules do not exert any force of attraction or repulsion on each other.
 - The collisions of molecules with one another and with walls of the vessel are perfectly elastic.
24. For accelerated motion, we have

$$u = 0, a = f \text{ and } s = s$$

$$\text{As we know that, } v^2 - u^2 = 2as,$$

$$\therefore v_1^2 - 0^2 = 2fs \Rightarrow v_1 = \sqrt{2fs}$$

$$\text{Distance travelled, } s_2 = v_1 t = t\sqrt{2fs}$$

For decelerated motion, we have

$$u = \sqrt{2fs}, a = -\frac{f}{2} \text{ and } v = 0$$

$$\text{As, } v^2 - u^2 = 2as$$

$$\therefore 0^2 - (\sqrt{2fs})^2 = 2 \times (-f/2) s_3$$

$$\text{Distance travelled, } s_3 = 2s$$

Also it is given that the total distance is 5s i.e., $s + s_2 + s_3 = 5s$

$$\Rightarrow s + t\sqrt{2fs} + 2s = 5s$$

$$\Rightarrow t\sqrt{2fs} = 2s$$

$$\Rightarrow s = \frac{1}{2}ft^2$$

Hence proved.

25. In equilibrium there is no motion of blocks i.e., the forces should balance out.

For block B, the equation is:

$$F = Mg \dots(i)$$

For block A the equation is:

$$F - mg \sin 30 = 0 \dots(ii)$$

Substituting the value of F from

$$Mg = mg \sin 30$$

$$\Rightarrow Mg = 100 \times 0.5 = 50 \text{ N}$$

So the weight of the block should be 50 N

Section C

26. Air is filled in the tyre adiabatically. let initial volume of air in tyre is V and after pumping one stroke volume become (V + dV) and pressure changes from P to (P + dP)

$$\text{By adiabatic equation } P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$P(V + dV)^\gamma = (P + dP)V^\gamma$$

$$PV^\gamma \left[1 + \frac{dV}{V}\right]^\gamma = P \left[1 + \frac{dP}{P}\right] V^\gamma \dots(1)$$

As volume of tyre V remains constant so dV/V is very small. By using binomial expansion in equation (1) we get

$$PV^\gamma \left[1 + \gamma \frac{dV}{V}\right] = PV^\gamma \left[1 + \frac{dP}{P}\right]$$

$$1 + \gamma \frac{dV}{V} = 1 + \frac{dP}{P}$$

$$\text{on solving we get } dV = \frac{V dP}{\gamma P}$$

Integrating both sides using limits W_1 to W_2 for work done and P_1 to P_2 for pressure we get

$$\int p dV = \int_{P_1}^{P_2} \frac{V dP}{\gamma}$$

$$\int_{W_1}^{W_2} dW = \frac{V}{\gamma} (P_2 - P_1) (V)$$

$$W_2 - W_1 = \frac{(P_2 - P_1)V}{\gamma}$$

27. If a is the acceleration of the whole system containing three blocks, then $F = \text{mass of the system} \times \text{acceleration of the system} =$

$$(m + m + m) \times a$$

$$\Rightarrow F = 3ma$$

$$\therefore a = \frac{F}{3m}$$

i. Net force on P,

$$F_1 = ma = m \times \frac{F}{3m}$$

$$\Rightarrow F_1 = \frac{F}{3}$$

ii. Force applied on Q by P = reaction force on P by Q,

$$F_2 = (m + m)a \text{ [since, the reaction force comes due to masses of Q and R]}$$

$$\therefore F_2 = 2m \times a = 2m \times \frac{F}{3m}$$

$$\therefore F_2 = \frac{2F}{3}$$



iii. Force applied on R by Q = reaction force on Q by R,

$$F_3 = m \times a = m \times \frac{F}{3m}$$

$$\therefore F_3 = \frac{F}{3}$$

28. As we know that,

The volume of water flowing per second

$$V = a_1 a_2 \sqrt{\frac{2h\rho_m g}{\rho(a_1^2 - a_2^2)}}$$

$$\therefore V = a_1 a_2 \sqrt{\frac{2hg}{a_1^2 - a_2^2}}$$

$$\therefore r_1 = 20\text{cm}, a_1 = \pi r_1^2 = \pi(20)^2 \text{cm}^2$$

$$r_2 = 10\text{cm}, a_2 = \pi r_2^2 = \pi(10)^2 \text{cm}^2$$

$$g = 980\text{cm/s}^2$$

$$\therefore V = \pi^2 (20)^2 (10)^2 \sqrt{\frac{2 \times 10 \times 980}{\pi^2 ((20)^4 - (10)^4)}} \text{ c.c./sec}$$

$$= \frac{175.93 \times 10^3}{\sqrt{15}} \text{ c.c./sec}$$

$$= \frac{175.93 \times 10^3}{\sqrt{15}} \times 60 \text{ c.c./min}$$

$$= 2728.7 \text{ litres/min}$$

OR

Volume of a bigger drop = n × Volume of a smaller droplet

$$\text{or } \frac{4}{3} \pi R^3 = n \times \frac{4}{3} \pi r^3 \text{ or } R^3 = nr^3$$

$$\text{or } R = n^{\frac{1}{3}} r$$

Terminal velocity of a small droplet is given by

$$v_1 = \frac{2}{9} \frac{r^2}{\eta} (\rho - \rho') g$$

Terminal velocity of a bigger drop is given by

$$v_2 = \frac{2}{9} \frac{R^2}{\eta} (\rho - \rho') g$$

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{v_2}{v_1} = \frac{R^2}{r^2}$$

$$\text{But } R = n^{\frac{1}{3}} r$$

$$\text{and } v_1 = 10 \frac{\text{cm}}{\text{s}}$$

$$v_2 = v_1 \times \left(\frac{R^2}{r^2} \right) = 10 \times \frac{n^{2/3} r^2}{r^2}$$

$$v_2 = 10 n^{\frac{2}{3}} \frac{\text{cm}}{\text{s}}$$

29. Consider two waves having same frequency, same nature travelling in same direction superimpose on each other. Let these waves be represented by

$$y_1(x, t) = A \sin(kx - \omega t) \dots(i)$$

$$\text{and } y_2(x, t) = A \sin(kx - \omega t + \phi) \dots(ii)$$

Here the second wave has been considered to be ahead in phase as compared to the first wave by a phase angle ϕ i.e., there is a phase difference of ϕ between the superposing waves.

Applying the superposition principle, we find that the resultant wave is the algebraic sum of two constituent waves.

Thus,

$$y(x, t) = y_1(x, t) + y_2(x, t)$$

$$= A \sin(kx - \omega t) + A \sin(kx - \omega t + \phi)$$

$$= 2A \left[\cos \frac{\phi}{2} \cdot \sin \left(kx - \omega t + \frac{\phi}{2} \right) \right]$$

$$\text{or } y(x, t) = 2A \cdot \cos \frac{\phi}{2} \cdot \sin \left(kx - \omega t + \frac{\phi}{2} \right) \dots(iii)$$

It shows that the resultant wave is also a sinusoidal wave of same frequency and wavelength travelling in +ve x-direction. But amplitude of resultant wave $A_r = 2A \cos \frac{\phi}{2}$ and its phase is $\frac{\phi}{2}$.

Two special cases are of particular interest here:

i. For constructive interference, $\phi = 0$, then $A_r = 2A \cos 0^\circ = 2A$ i.e Maximum Amplitude

ii. If $\phi = \pi$, then $A_r = 2A \cos \left(\frac{\pi}{2} \right) = 0$ and the amplitude of the resultant wave is zero. It is destructive interference.

OR

Ultrasonic beep frequency emitted by the bat, $v = 40 \text{ kHz}$

Velocity of the bat, $v_b = 0.03 v$

Where, $v =$ velocity of sound in air

The apparent frequency of the sound striking the wall is given as:

$$\begin{aligned}
 v' &= \left(\frac{v}{v-v_b} \right) v \\
 &= \left(\frac{v}{v-0.03v} \right) 40 \\
 &= \frac{40}{0.97} \text{ kHz}
 \end{aligned}$$

This frequency is reflected by the stationary wall ($v_s = 0$) toward the bat.

The frequency (v'') of the received sound is given by the relation:

$$\begin{aligned}
 v'' &= \left(\frac{v}{v+v_s} \right) v' \\
 &= \left(\frac{v+0.3v}{v} \right) \times \frac{40}{0.97} \\
 &= \frac{1.03 \times 40}{0.97} = 42.47 \text{ kHz}
 \end{aligned}$$

30. Triple point of water on absolute scale A, $T_1 = 200 \text{ A}$

Triple point of water on absolute scale B, $T_2 = 350 \text{ B}$

Triple point of water on Kelvin scale, $T_K = 273.15 \text{ K}$

The temperature 273.15 K on Kelvin scale, T_K is equivalent to 200 A on absolute scale A, T_1 .

i.e. $T_1 = T_K$

$\Rightarrow 200 \text{ A} = 273.15 \text{ K}$

$$\therefore A = \frac{273.15}{200}$$

The temperature 273.15 K on Kelvin scale, T_K is equivalent to 350 B on absolute scale B, T_2 .

i.e. $T_2 = T_K$

$\Rightarrow 350 \text{ B} = 273.15$

$$\therefore B = \frac{273.15}{350}$$

Now as T_A is triple point of water on scale A and

T_B is triple point of water on scale B.

$$A \times T_A = B \times T_B$$

$$\Rightarrow \frac{273.15}{200} \times T_A = \frac{273.15}{350} \times T_B$$

$$\therefore T_A = \frac{200}{350} T_B$$

$\Rightarrow T_A = \frac{4T_B}{7}$, this is the required relation between the triple points on the mentioned two scales of temperature.

Section D

31. For calculation of this problem we can proceed in following manner-

Area of cross-section of the U-tube = A

Density of the mercury column = ρ

Acceleration due to gravity = g

Restoring force, $F =$ Weight of the mercury column of a certain height

$$F = -(\text{Volume} \times \text{Density} \times g)$$

$$F = -(A \times 2h \times \rho \times g) = -2\rho gh = -k \times \text{Displacement in one of the arms (h)}$$

Where, $2h$ is the height of the mercury column in the two arms k is a constant, given by $k = -\frac{F}{h} = 2A\rho g$

$$\text{Time period, } T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{m}{2A\rho g}}$$

Where, m is the mass of the mercury column

Let l be the length of the total mercury in the U-tube.

Mass of mercury, $m =$ Volume of mercury \times Density of mercury

$$= Al\rho$$

$$\therefore T = 2\pi \sqrt{\frac{m}{2A\rho g}} = 2\pi \sqrt{\frac{l}{2g}}$$

Hence, the mercury column executes simple harmonic motion with time period $2\pi \sqrt{\frac{l}{2g}}$.

OR

The potential energy (PE) of a simple harmonic oscillator is

$$PE = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 x^2 \dots (i)$$

When, PE is plotted against displacement x, we will obtain a parabola.

When $x = 0$, $PE = 0$

When $x = \pm A$, $PE = \text{maximum}$

$$= \frac{1}{2}m\omega^2 A^2$$

The kinetic energy (KE) of a simple harmonic oscillator $KE = \frac{1}{2}mv^2$

But velocity of oscillator $v = \omega\sqrt{A^2 - x^2}$

$$\Rightarrow KE = \frac{1}{2}m[\omega\sqrt{A^2 - x^2}]^2$$

$$\text{or } KE = \frac{1}{2}m\omega^2 (A^2 - x^2) \dots (ii)$$

This is also parabola, if we plot KE against displacement x

i.e. $KE = 0$ at $x = \pm A$

and $KE = \frac{1}{2}m\omega^2 A^2$ at $x = 0$

Now, total energy of the simple harmonic oscillator = PE + KE [using Eqs. (i) and (ii)]

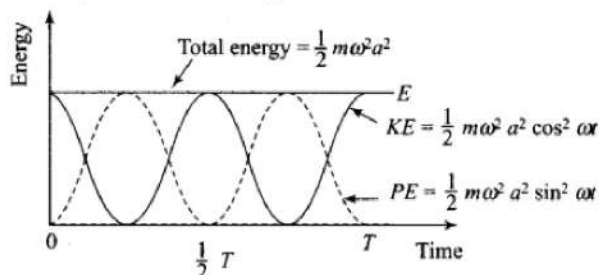
$$= \frac{1}{2}m\omega^2 x^2 + \frac{1}{2}m\omega^2 (A^2 - x^2)$$

$$= \frac{1}{2}m\omega^2 x^2 + \frac{1}{2}m\omega^2 A^2 - \frac{1}{2}m\omega^2 x^2$$

$$TE = \frac{1}{2}m\omega^2 A^2 = \text{constant}$$

which is a constant and independent of x.

Plotting under the above guidelines KE, PE and TE versus displacement x-graph as follows:



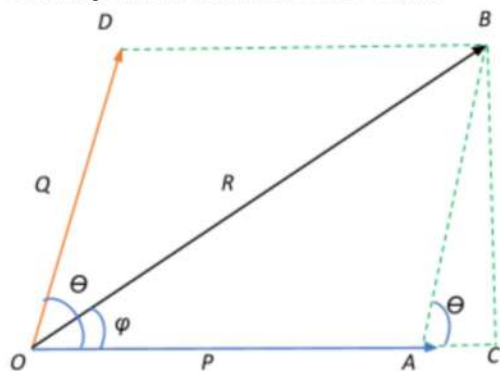
Important point: From the graph, we note that potential energy or kinetic energy completes two vibrations in a time during which S.H.M. completes one vibration.

Thus the frequency of potential energy or kinetic energy is double that of S.H.M.

32. Let us begin with the parallelogram law of vector addition.

The law states that if two vector quantities are represented by two adjacent sides of a parallelogram then the resultant of these two vectors will be the diagonal of the parallelogram.

Let us try and understand how this works:



Let the two vectors \vec{P} and \vec{Q} acting from the same point O be represented both in magnitude and direction as two adjacent sides OA and OD of a parallelogram OABD.

Let the angle between the two vectors be θ .

According to our definition of the parallelogram law of vector addition, the diagonal of the parallelogram OB represents the resultant of \vec{P} and \vec{Q} . Thus, let the resultant of the two vectors be represented by \vec{R} that is at an angle ϕ with \vec{P} .

$$\vec{R} = \vec{P} + \vec{Q}$$

The sides $AB = OD = |Q|$ and $OA = BD = |P|$ and diagonal $OB = |R|$

Consider the right angle triangle OBC. From the Pythagorean theorem.

$$OB^2 = OC^2 + BC^2 = (OA + AC)^2 + BC^2$$



Consider the triangle ABC:

$$\cos \theta = \frac{AC}{AB} \Rightarrow AC = AB \cos \theta = OD \cos \theta \Rightarrow AC = |Q| \cos \theta$$

$$\text{Also } \sin \theta = \frac{BC}{AB} \Rightarrow BC = AB \sin \theta = OD \sin \theta \Rightarrow BC = |Q| \sin \theta$$

Substituting the AC and BC expressions in the OB^2 Pythagorean theorem, we get:

$$|R|^2 = (|P| + |Q| \cos \theta)^2 + (|Q| \sin \theta)^2 = |P|^2 + 2|P||Q| \cos \theta + |Q|^2 (\cos^2 \theta + \sin^2 \theta) = |P|^2 + 2|P||Q| \cos \theta + |Q|^2$$

Therefore, the magnitude of the resultant

$$|R| = \sqrt{|P|^2 + 2|P||Q| \cos \theta + |Q|^2}$$

Now let us find the direction of the resultant:

$$\text{From triangle ABC } \tan \phi = \frac{BC}{OC} = \frac{BC}{OA+AC} = \frac{|Q| \sin \theta}{|P|+|Q| \cos \theta}$$

Therefore, the direction of the resultant with respect to \vec{P} is given by

$$\phi = \tan^{-1} \left(\frac{|Q| \sin \theta}{|P|+|Q| \cos \theta} \right)$$

We have thus obtained the magnitude and direction of the resultant \vec{R}

Now, when $\theta = 0^\circ$:

$$|R| = \sqrt{|P|^2 + 2|P||Q| \cos 0 + |Q|^2} = \sqrt{|P|^2 + 2|P||Q| + |Q|^2} = \sqrt{(|P| + |Q|)^2} \Rightarrow |R| = |P| + |Q|$$

$$\phi = \tan^{-1} \left(\frac{|Q| \sin 0}{|P|+|Q| \cos 0} \right) = \tan^{-1}(0) = 0$$

Thus, in this case, the magnitude of the resultant vector will be the sum of the magnitudes of the adjacent vectors and the resultant lies in the direction of \vec{P}

And finally, when $\theta = 90^\circ$:

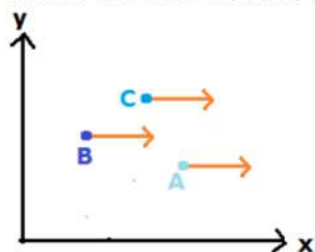
$$|R| = \sqrt{|P|^2 + 2|P||Q| \cos 90 + |Q|^2} = \sqrt{|P|^2 + |Q|^2} = \sqrt{(|P|^2 + |Q|^2)}$$

$$\phi = \tan^{-1} \left(\frac{|Q| \sin 90}{|P|+|Q| \cos 90} \right) = \tan^{-1} \left(\frac{Q}{P} \right)$$

(since $\sin 0^\circ = 0$, $\cos 0^\circ = 1$, $\sin 90^\circ = 1$, $\cos 90^\circ = 0$, $\tan 0^\circ = 0$)

OR

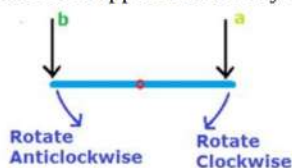
A vector has magnitude and direction but in general, it does not have a fixed location of space because a vector can be translated parallel to itself or we can say if a vector is moved parallel to itself keeping its direction and magnitude same then the vector is same or there is no effect on vector as can be shown in figure.



Now here there are three vectors at A, B, C all have the same length so have the same magnitude, have the same direction towards positive x-axis and are thus parallel to each other so all the three vectors are same so there is no effect of location i.e. position is not fixed but in case of position vector position of each point is different and position vector denotes the position in terms of coordinates of x and y so position vector has a fixed location and are also directed from the origin so two position vectors cannot be parallel if they are denoting different positions so, position vector has a definite position in space but in general, all vectors does not have a specific position in space

Yes, the vector can certainly vary with time and many vector quantities are just rate of variation of other quantities or vectors can be a function of time for e.g. Velocity of a particle in uniform motion is a function of time and as the time increases the velocity of particle increases or decreases depending upon the acceleration of particle i.e. velocity changes with time likewise in a general vector can vary with time

Now we have two equal vectors a and b at different locations in space they necessarily need not have same physical effects though in specific cases they can have some physical effects this is not true always for e.g. two forces of the same magnitude and same direction applied on a body-fixed lever can produce different turning effects as shown in figure



As can be seen, both a and b are directed vertically downwards i.e. the same direction and have the same magnitude so both are equal but turning effect will be different due to them because their location is different in same so we conclude that equal vectors a and b at different locations in space do not necessarily have identical physical effects

33. Radii of the ring and the disc, $r = 10 \text{ cm} = 0.1 \text{ m}$

Initial angular speed, $\omega_z = 10 \pi \text{ rad s}^{-1}$

Coefficient of kinetic friction, $\mu_k = 0.2$

Initial velocity of both the objects, $u = 0$

Motion of the two objects is caused by frictional force. As per Newton's second law of motion, we have frictional force, $f = ma$

$$\mu_k mg = ma$$

Where,

a = Acceleration produced in the objects

m = Mass

$$\therefore a = \mu_k g \dots (i)$$

As per the first equation of motion, the final velocity of the objects can be obtained as:

$$v = u + at$$

$$= 0 + \mu_k gt$$

$$= \mu_k gt \dots (ii)$$

The torque applied by the frictional force will act in a perpendicularly outward direction and cause a reduction in the initial angular speed.

$$\text{Torque, } T = -I\alpha$$

α = Angular acceleration

$$u_z mgr = -I\alpha$$

$$\therefore a = \frac{-\mu_k mgr}{I} \dots \dots (iii)$$

Using the first equation of rotational motion to obtain the final angular speed:

$$\omega = \omega_e + at$$

$$= \omega_x + \frac{-\mu_k mgr}{I} t \dots \dots (iv)$$

Rolling starts when linear velocity, $v = ru$

$$\therefore v = r \left(\omega_0 - \frac{\mu_k grt}{I} \right) \dots \dots (v)$$

Equating equations (ii) and (v), we get:

$$\mu_k gt = r \left(\omega_0 - \frac{\mu_k grt}{I} \right)$$

$$= r\omega_0 - \frac{\mu_k gr^2 t}{I} \dots \dots (vi)$$

For the ring $I = mr^2$

$$\therefore \mu_k gt = r\omega_0 - \frac{\mu_k gr^2 t}{mr^2}$$

$$= r\omega_0 - \frac{\mu_k gr^2 t}{mr^2}$$

$$2\mu_k gt = r\omega_0$$

$$\therefore t_r = \frac{r\omega_0}{2\mu_k g}$$

$$= \frac{0.1 \times 10 \times 3.14}{2 \times 0.2 \times 9.8} = 0.80 \text{ s} \dots \dots (vii)$$

For the ring $I = \frac{1}{2}mr^2$

$$\therefore \mu_k gt_d = r\omega_0 - \frac{\mu_k gr^2 t}{\frac{1}{2}mr^2}$$

$$= r\omega_0 - 2\mu_k gt$$

$$3\mu_k gt_d = r\omega_0$$

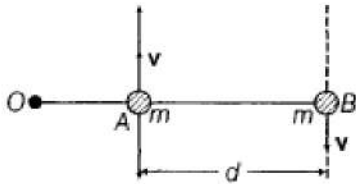
$$\therefore t_d = \frac{r\omega_0}{3\mu_k g}$$

$$= \frac{0.1 \times 10 \times 3.14}{3 \times 0.2 \times 9.8} = 0.53 \text{ s} \dots \dots (viii)$$

Since $t_d > t_r$, the disc will start rolling before the ring.

OR

Suppose, O be the origin chosen.



Then, angular momentum of particle at A is

$$I_1 = OA \times p = OA \times mv$$

$$= m(OA \times v)$$

and angular momentum of particle at B is

$$I_2 = OB \times p = OB \times (-mv)$$

$$= -m(OB \times v)$$

so, total angular momentum of the system of particles is

$$L = I_1 + I_2$$

$$= m(OA \times v) - m(OB \times v)$$

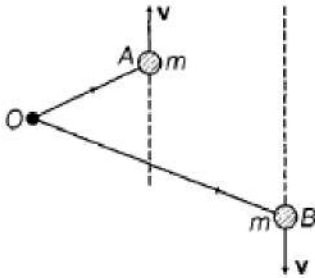
$$= m(OA - OB) \times v$$

$$= m(BA) \times v$$

$$= m(BA) \times v$$

{As, BA = position vector of A - position vector of B}

Above expression is independent of choice of origin.



This is true even when particles are not in a straight line.

$$L_i(I_1 + I_2 = m(OA \times v - OB \times v)$$

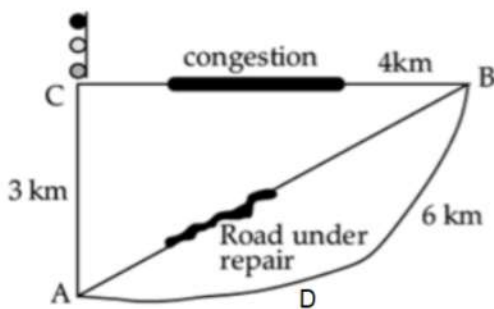
$$= m(BA) \times v$$

Which is the same as a previous result. So, the angular momentum of the system is independent of the choice of origin.

Section E

34. Read the text carefully and answer the questions:

Tabu lives at A. He was supposed to do to his uncle's house at B. A and B is connected by a straight road 5 km long. But that day the road was under repair. So, all the buses were following a diversion via C. A to B via C is 7 km. Moreover, this route is congested There is a traffic signal at C also.



Tabu got a seat just behind the driver. He noticed that the minimum reading on the speedometer was 15km/h. But ultimately the bus took 1 hour to each B. He could not understand the fallacy.

(i) Distance is the actual path covered i.e., $3 + 4 = 7$ km.

Displacement is the straight line distance from A to B i.e., $\sqrt{3^2 + 4^2} = 5$ km.

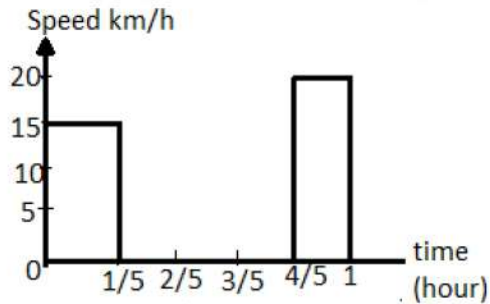
(ii) Halt timing at the traffic signal, slow speed at the congested areas and halt-timing at the bus stops are also to be taken into account.

(iii) Average speed = $\frac{\text{Total distance traversed}}{\text{Total time taken}}$

$$= \frac{6}{1} = 6 \text{ km/h}$$

OR

Following graph shows the required v-t graph.

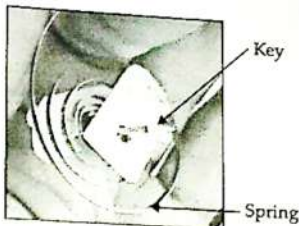


35. Read the text carefully and answer the questions:

Clockwork refers to the inner workings of mechanical clock or watch (where it is known as "movement") and different types of toys which work using a series of gears driven by a spring. Clockwork device is completely mechanical and its essential parts are:

- A key (or crown) which you wind to add energy
- A spiral spring in which the energy is stored
- A set of gears through which the spring's energy is released. The gears control how quickly (or slowly) a clockwork machine can do things. Such as in mechanical clock/watch the mechanism is the set of hands that sweep around the dial to tell the time. In a clockwork car toy, the gears drive the wheels.

Winding the clockwork with the key means tightening a sturdy metal spring, called the mainspring. It is the process of storing potential energy. Clockwork springs are usually twists of thick steel, so tightening them (forcing the spring to occupy a much smaller space) is actually quite hard work. With each turn of the key, fingers do work and potential energy is stored in the spring. The amount of energy stored depends on the size and tension of the spring. Harder a spring is to turn and longer it is wound, the more energy it stores.



While the spring uncoils, the potential energy is converted into kinetic energy through gears, cams, cranks and shafts which allow wheels to move faster or slower. In an ancient clock, gears transform the speed of a rotating shaft so that it drives the second hand at one speed, the minute hand at $\frac{1}{60}$ of that speed, and the hour hand at $\frac{1}{3600}$ of that speed. Clockwork toy cars often use gears to make themselves race along at surprising speed.

- (i) Winding the spring means tightening a sturdy metal spring. It is the process of storing potential energy (forcing the spring to occupy a much smaller space) is actually quite hard work. With each turn of the key, fingers do work and potential energy is stored in the spring.
- (ii) When the spring uncoils, the potential energy is converted into kinetic energy through gears, cams, cranks and shafts which allow wheels to move faster or slower.
- (iii) In an ancient clock, gears transform the speed of a rotating shaft so that it drives the second hand at one speed, the minute hand at $\frac{1}{60}$ of that speed, and the hour hand at $\frac{1}{3600}$ of that speed. Clockwork toy cars often use gears to make themselves race along at surprising speed.

OR

The amount of energy stored depends on the size and tension of the spring. Harder a spring is to turn and longer it is wound, the more energy it stores.

